SOME CONFIGURATIONS OF ISENTROPIC DECOMPOSITIONS OF TWO-DIMENSIONAL DISCONTINUITIES

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The solution of some two-dimensional nonstationary problems on the motion of two plane pistons in a polytropic gas are constructed.

1. Let the polytropic gas with the equation of state $p = a^2\rho^\gamma$ (p is the pressure, ρ the density, γ the adiabatic index, $a^2 =$ const) be at rest at the initial instant $t = 0$ within some dihedral angle formed by the two intersecting planes P_1 and P_2, the angle α between them satisfying the relation $0 < \alpha \leqslant \frac{1}{2}\pi$. We shall consider the problem of determining the nonstationary plane flows which arise in the gas when the planes P_1 and P_2, which play the role of pistons, begin at the instant $t = 0$ to move out of the gas at the constant velocities v_1 and v_2, respectively. The resulting flows are two-dimensional and self-similar, so that the components u_1 and u_2 of the velocity vector and the acoustic speed C which are to be determined depend on the two independent self-similar variables $\xi_1 = x_1 / t$, $\xi_2 = x_2 / t$, where x_1 and x_2 are plane Cartesian coordinates. We assume here that the flows are free of shock waves and contact discontinuities, and are therefore isentropic and potential. Their potentiality follows from Thomson's theorem, which is valid in this case by virtue of the fact that the flows contain weak discontinuities only.

In the case where the planes P_1 and P_2 begin to withdraw according to an arbitrary law, the solution of the problem can be sought in the class of double waves. The authors of [1] solved the problem of the motion of two mutually perpendicular pistons according to an arbitrary law in an isothermal gas in the class of double waves. They also formulated the Goursat problem for the double wave equation for the motion of two pistons in a polytropic gas. Solution of the Goursat problem alone, however, generally does not permit construction of a complete picture of the motion even with the simplest laws of piston motion. This is because the domains of definition of the Goursat problem usually do not coincide with the natural domains of definition of the flows in either the physical space x_1, x_2 t, or in the hodograph plane, and comprise but a portion of the latter. It is necessary therefore, to pose additional problems in order to fill out the entire domain of flow definition. The present paper is devoted precisely to the posing of such supplementary problems and to the study of possible flow configurations arising due to the specific discontinuity decomposition which occurs when the pistons begin to move with constant velocities. The flow region here consists of the regions of the double self-similar waves and simple waves, and the regions of constant motion. The Goursat problem and the mixed problems for the double wave equation are solved numerically by the method of characteristics as long as the double wave equation is of the hyperbolic type.

The use of the method of characteristics in the hyperbolic case permits complete solution of several problems on the motion of pistons when a vacuum zone arises in the region adjacent to the line where the planes P_1 and P_2 intersect. A vacuum zone may not arise in the general case with small velocities V_1 and V_2 (as compared with acoustic speed in the unperturbed gas), however, in which case a line of parabolicity of the double wave equation and beyond it a region of ellipticity of this equation generally arise in the neighborhood of the line l In the present paper, we shall concern ourselves only with the regions where the equation under consideration is hyperbolic.

The particular case of our problem where one of the planes P_1, P_2 remains motionless while the other is moving with an infinite velocity (efflux into a vacuum) is considered in [2]. Analogous particular problems for three-dimensional self-similar flow are studied in [3]. The problem of uniqueness of the solutions will not be dealt with.

2. Let us consider the problem of contiguity of flows of the double and simple wave type, and some of the properties of flows in the event of such contiguity which we shall need below. The systems of equations describing simple and double waves for the self-similar case (see [4 and 5]) can be written as

$$u_1'^2 + u_2'^2 = 1, \qquad u_1'\xi_1 + u_2'\xi_2 - \left(\frac{\gamma-1}{2}\,\theta + u_1u_1' + u_2u_2'\right) = 0 \tag{2.1}$$

for simple waves, and as

$$\frac{\gamma-1}{2}\,\theta\,[(1-\theta_1^2)\,\theta_{22} + 2\theta_1\theta_2\theta_{12} + (1-\theta_2^2)\theta_{11}] + \frac{\gamma-3}{2}\,(\theta_1^2+\theta_2^2) + 2 = 0 \tag{2.2}$$

$$\xi_i = u_i + \frac{\gamma-1}{2}\,\theta\theta_i \qquad (i=1,\,2) \tag{2.3}$$

for double waves.

Here

$$u_i = u_i(\theta), \quad \theta = \frac{2}{\gamma-1}\,C, \quad \theta_i = \frac{\partial\theta}{\partial u_i}, \quad \theta_{ik} = \frac{\partial^2\theta}{\partial u_i\partial u_k}$$

the prime indicating differentiation with respect to θ.

Simple waves in the hodograph plane u_1, u_2 have a certain corresponding curve $\Psi(u_1, u_2) = 0$; double waves have a corresponding region S in which the function $\theta = \theta(u_1, u_2)$ is defined.

The category to which Equation (2.2) belongs is determined by the sign of the expression $R = \theta_1^2 + \theta_2^2 - 1$. For $R > 0$, Equation (2.2) is of the hyperbolic type.

The equations of the characteristic strip for (2.2) are

$$(1-\theta_1^2)du_1^2 - 2\theta_1\theta_2du_1du_2 + (1-\theta_2^2)\,du_2^2 = 0 \tag{2.4}$$

$$d\theta = \theta_1du_1 + \theta_2du_2 \tag{2.5}$$

$$(1-\theta_1^2)\,d\theta_2du_1 + (1-\theta_2^2)\,d\theta_1du_2 + \frac{(\gamma-3)(\theta_1^2+\theta_2^2)+4}{(\gamma-1)\,\theta}\,du_1du_2 = 0 \tag{2.6}$$

P r o p e r t y 2.1. If the curve $\Psi(u_1, u_2) = 0$ in the hodograph plane corresponds to some simple wave which is contiguous to the double wave region along it, then characteristic equation (2.4) is satisfied along this curve.

This property follows from the relation

$$(\theta_1du_1 + \theta_2du_2)^2 = du_1^2 + du_2^2. \tag{2.7}$$

which is a consequence of (2.1), (2.5), since the function θ is continuous in passing through the curve $\Psi(u_1, u_2) = 0$.

We note that in the case of an arbitrary simple wave, the existence of θ_1 and θ_2 such that condition (2.6) is fulfilled cannot be guaranteed for the entire specified curve $\Psi(u_1, u_2) = 0$.

In fact, let us fix in the plane ξ_1, ξ_2 some point ξ_1°, ξ_2° with specified u_1°, u_2°, Θ° through which the line separating the simple and double wave regions passes. Then, replacing Θ_1 by Θ_2 in accordance with (2.5), we obtain from (2.6) the following ordinary nonlinear differential equation for the function Θ :

$$2(\theta_2-\varphi)\theta_2'+(\theta_2^2-1)(\theta_2-\varphi)\frac{\varphi'}{1-\varphi^2}+$$

$$+\frac{1}{(\gamma-1)\theta}[(\gamma-3)(1+\theta_2^2-2\varphi\theta_2)+4(1-\varphi^2)]=0$$

Here Θ acts as the independent variable, the initial data obtained from (2.3) are of the form $\Theta_2(\theta^\circ)=\Theta_2^\circ$, and $u_2'=\varphi(\Theta)$ is an arbitrary function.

Thus, it can generally be asserted that Θ_2 is defined only in some neighborhood of the point Θ°. However, as we shall see below, in the case of our two-piston problem the simple waves to which the double wave must be made contiguous are of special form, and characteristic strip condition (2.6) is fulfilled for all Θ , i.e. along the entire line $\Psi(u_1, u_2)=0$.

We note that Formula (2.7) implies the inverse property: any characteristic of Equation (2.2) corresponds to some simple wave.

P r o p e r t y 2.2. If the curve $\Psi(u_1, u_2)$ corresponds to a simple wave, being the first-family characteristic for the double wave equation, then the equipotential lines of the principal quantities in the simple wave (straight lines in the plane $\xi_1\xi_2$ (2.1)) touch the second-family characteristics at the points of the contiguity curve which corresponds to the curve $\Psi(u_1, u_2)=0$ in the plane $\xi_1\xi_2$.

Parametrically, the second-family characteristics are given by Equations (2.3), in which u_1 and u_2 are replaced by their corresponding expressions in terms of Θ . Let the vectors $(\delta u_1, \delta u_2)$ and (du_1, du_2) determine the directions of the tangents to the characteristics of the first and second families in the plane u_1, u_2, respectively. Expressions (2.1) imply that in order to prove the foregoing property it is sufficient to verify the relation

$$\Gamma=\delta u_1 d\xi_1+\delta u_2 d\xi_2=0 \qquad (2.8)$$

where the differentials $d\xi_1$, $d\xi_2$ correspond to the second-family characteristics. Representing Γ as

$$\Gamma=\delta u_1\left(du_1+\frac{\gamma-1}{2}\theta_1 d\theta+\frac{\gamma-1}{2}\theta d\theta_1\right)+\delta u_2\left(du_2+\frac{\gamma-1}{2}\theta_2 d\theta+\frac{\gamma-1}{2}\theta d\theta_2\right)$$

and making use of relations (2.4) to (2.6) and Formula

$$\frac{\delta u_1}{\delta u_2}\frac{du_1}{du_2}=\frac{1-\theta_2^2}{1-\theta_1^2}$$

which follows from (2.4), we finally obtain

$$\Gamma=\frac{\delta u_2}{du_1}\frac{\gamma-1}{2}\frac{\theta_1\theta_2}{1-\theta_1^2}[(1-\theta_1^2)du_1^2+(1-\theta_2^2)du_2^2-2\theta_1\theta_2\,du_1du_2]=0$$

P r o p e r t y 2.3 . If the double wave is contiguous with a one-dimensional simple Rieman wave of the form

$$u_1=\alpha_1\theta+\beta_1, \qquad u_2=\alpha_2\theta+\beta_2 \qquad (2.9)$$

where α_i and β_i are constant and $\alpha_1^2+\alpha_2^2=1$ (by a rotation the coordinate axes, this case readily reduces to that where, for example, $u_2\equiv 0$), then the line of contiguity is analytically determinate in the plane $\xi_1\xi_2$.

In fact, making use of the relations for $\alpha_1\neq 0$ and $\alpha_2\neq 0$ which follow from (2.9),

$$\frac{du_1}{du_2}=\frac{\alpha_1}{\alpha_2}, \qquad \alpha_1\theta_1+\alpha_2\theta_2=1, \qquad \frac{d\theta_1}{d\theta_2}=-\frac{\alpha_2}{\alpha_1} \qquad (2.10)$$

we reduce Equation (2.6) to the form

$$d\Theta_2 \frac{\alpha_1^2 - (1-\alpha_2\Theta_2)^2 - \alpha_2^2(1-\Theta_2^2)}{(\gamma-3)[\alpha_1^2\Theta_2^2 + (1-\alpha_2\Theta_2)^2] + 4\alpha^2_1} + \frac{\alpha_2}{\gamma-1}\frac{d\Theta}{\Theta} = 0 \tag{2.11}$$

Integrating (2.11), we obtain

$$(\gamma-3)\left[\Theta_2^2 + \left(\frac{1-\alpha_2\Theta_2}{\alpha_1}\right)^2\right] + 4 + C\Theta^{-\frac{\gamma-3}{\gamma-1}} = 0 \qquad (\gamma \neq 3) \tag{2.12}$$

$$\frac{1}{2\alpha_1^2}\Theta_2^2 - \frac{\alpha_2}{\alpha_1^2}\Theta_2 + \ln\Theta + C = 0 \qquad (\gamma = 3) \tag{2.13}$$

where C is an integration constant determined from the conditions of the problem. Formulas (2.3) then immediately imply the equations giving the line of contiguity in parameteric form

$$\xi_i = \alpha_i\Theta + \beta_i + \frac{\gamma-1}{2}\Theta\Theta_i \tag{2.14}$$

where Θ_i must be replaced by their expressions in terms of Θ in accordance with Formulas (2.10) and (2.12), and where Θ acts as a parameter.

3. Let us now describe the method of solving the problem stated in the introduction.

We first consider the question as to the conditions which the function $\Theta(u_1, u_2)$ describing the double wave must satisfy at the movable wall (piston). Let the equation of motion of the rectilinear movable wall at the coordinates ξ_1, ξ_2 be of the form

$$a_1\xi_1 + a_2\xi_2 + a_3 = 0, \qquad a_i = \text{const} \tag{3.1}$$

Its normal velocity is $|a_3|/\sqrt{a_1^2 + a_2^2}$. From the condition of no gas flow through the wall we have

$$a_1u_1 + a_2u_2 + a_3 = 0 \tag{3.2}$$

Substituting ξ_i according to Formulas (2.3) into (3.1), we have the condition

$$a_1\Theta_1 + a_2\Theta_2 = 0 \tag{3.3}$$

for the function Θ in the hodograph plane along the line (3.1).

Let us consider the region of interference of the simple Riemann waves (Fig.1) which arises upon withdrawal of the two plane pistons, the angle α between which is acute, at the constant velocities V_1 and V_2. The acoustic speed C_0 in the unperturbed gas which prior to the beginning of motion occupies a dihedral angle bounded by the planes $x_1 = 0$ and $x_2 = x_1 \cot\alpha$ at $t = 0$ is assumed to be 1. It is therefore necessary to consider the case $0 < V_i \leq 2/(\gamma-1)$, since the case $V_i > 2/(\gamma-1)$ $(i = 1, 2)$ coincides with the case $V_i = 2/(\gamma-1)$, and leads to the problem of gas efflux into the vacuum from the dihedral angle whose walls are instantaneously at the instant $t = 0$.

It is clear that at sufficient distances away from the line where the pistons intersect, when $t = t_0$ and $V_i < 2/(\gamma-1)$, one-dimensional motions will occur near the moving walls, and that the walls will be contiguous with the steady-flow regions which are contiguous with the Riemann wave regions at the weak discontinuities (the lines DF and D_1F_1). The Riemann discharge waves, here self-similar, will, in turn, be contiguous with the gas at rest at the second weak discontinuities (the lines EM and EM_1). The equations of the movable walls OC and OC_1 in the coordinates ξ_1 and ξ_2 are then

$$\xi_1 = -V_1, \quad \cos\alpha\xi_1 - \sin\alpha\xi_2 - V_2 = 0 \tag{3.4}$$

The equations of the lines DE and D_1E can be written in explicit form (Property 2.3). It is true that in order to find the equation of the curve DE, it is necessary instead integrating Equation (2.6), which is in this case identically fulfilled along $DE(\Theta_1 = 1, u_2 = 0)$, to integrate Equation (2.2) directly. The latter reduces to an ordinary equation on DE by virtue of the relation

$$d\Theta_1 = \Theta_{11}du_1 + \Theta_{12}du_2$$

Finally, for the curve DE we have the parametric equations (for $\gamma \neq 3$)

$$\xi_1 = \theta - \frac{2}{\gamma-1} + \frac{\gamma-1}{2}\theta, \qquad \xi_2 = \frac{\gamma-1}{2}\theta\left(C_\alpha \theta^{\frac{3-\gamma}{\gamma-1}} - \frac{\gamma+1}{\gamma-3}\right)^{1/2} \tag{3.5}$$

where the integration constant

$$C_\alpha = \left(\frac{\gamma+1}{\gamma-3} + \cot^2\frac{\alpha}{2}\right)\left(\frac{2}{\gamma-1}\right)^{\frac{\gamma-3}{\gamma-1}} \tag{3.6}$$

is determined from the condition of passage of the integral curve throu h the point E (1, cot $\frac{1}{2}\alpha$) lying on the bisector of the angle COC_1. Here

$$\theta = 2/(\gamma-1) \text{ at the point } E, \quad \theta = 2/(\gamma-1) - V_1 \text{ at the point } D$$

In exactly the same way, for $\gamma = 3$ we integrate (2.2) along DE to obtain

$$\xi_1 = 2\theta - 1, \qquad \xi_2 = \theta\sqrt{\cot^2(\alpha/2) - 2\ln\theta} \tag{3.7}$$

Analogous equations which follow from Formulas (2.12) to (2.14) can also be written for the curve D_1E . The radicand in Formula (3.5) for

$$\frac{2}{\gamma-1} - V_1 \leqslant \theta \leqslant \frac{2}{\gamma-1}$$

and in Formula (3.7) for any $0 < \alpha \leq \frac{1}{2}\pi$ is positive, so that the double waves can be contiguous with the simple Riemann wave along all of DE (the same applies to D_1E).

In Fig. 1 (and then in Figs. 3, 5, 7) the regions denoted by the number (1) correspond to the regions of steady flow or rest, those denoted by the number (2) to simple wave regions, and (3) to double wave regions, For the problem under consideration, we always construct a solution in which regions of the type (1) are contiguous with regions of the type (2), regions of the type (2) are contiguous with regions of the type (3), but regions of the type (3) are not directly contiguous with regions of the type (1). Then, in accordance with Property 2.2, the lines CD and C_1D_1 are straight lines tangent to the curves DE and D_1E at the points D and D_1 .

In the region $dDED_1d_1$, the Goursat problem for equation (2.2) must be solved with data on the characteristics DE and D_1E. In regions (3) touching the points C and C_1, it is necessary to solve mixed problems with data on the characteristics CS and C_1S_1 and conditions of the type (3.3) at

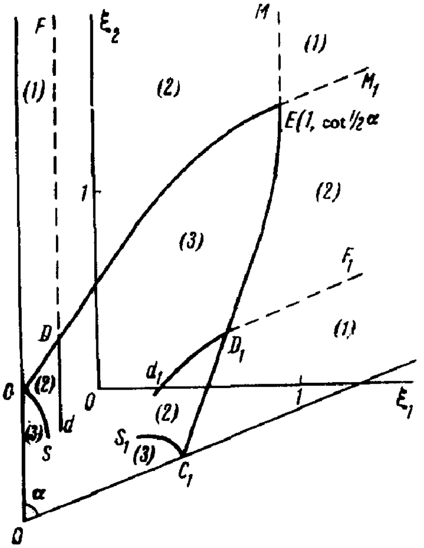

Fig. 1

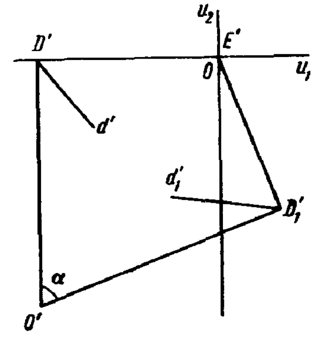

Fig. 2

the walls OC and OC_1,

The form of the characteristics CS and C_1S_1 is determined after solving the Goursat problen and constructing simple waves in the regions $SCDd$ and $S_1C_1D_1d_1$ by means of the solution of the Cauchy problem for ordinary equation (2.6) with the initial data $\Theta_1=\Theta_1^\circ$, $\Theta_2=\Theta_2^\circ$, respectively, at the points C and C_1.

We shall show that

$$\Theta_1^{\circ 2}+\Theta_2^{\circ 2}>1 \quad \text{for } V_i\leqslant 2/(\gamma-1), \quad \gamma>1, \quad 0<\alpha\leqslant {}^1/_2\pi \tag{3.8}$$

in all cases.

Inequality (3.8) need only be proved for the point C and arbitrary V_1, γ, and α from the indicated region (condition (3.8) will then be fulfilled at the point C_1, since the point E lies on the bisector of the angle COC_1).

The projections of the vector $\tau=(\tau_1,\tau_2)$ tangent to the curve DE at the point D can then be written as

$$\tau_1=\frac{\gamma+1}{2}, \quad \tau_2=\frac{\gamma+1}{4}\frac{(\gamma-3)\varepsilon-2(\gamma-1)}{\sqrt{(\gamma-3)[(\gamma-3)\varepsilon-(\gamma+1)]}} \qquad (\gamma\neq 3) \tag{3.9}$$

$$\varepsilon=\left(\frac{\gamma+1}{\gamma-3}+\cot^2\frac{\alpha}{2}\right)\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}} \tag{3.10}$$

$$\tau_1=2, \quad \tau_2=\frac{\cot^2(\alpha/2)-2\ln(1-V_1)-1}{\sqrt{\cot^2(\alpha/2)-2\ln(1-V_1)}} \qquad (\gamma=3) \tag{3.11}$$

by virtue of equations (3.5) and (3.7).

Along the straight line OC, $\Theta_1=0$. Constructing the straight line DC through the point D in the direction of the vector τ, in accordance with Formulas (2.3) at the point C we have

$$\Theta_1^0=0, \quad \Theta_2^0=\frac{(\gamma-3)\varepsilon-4}{2\sqrt{(\gamma-3)[(\gamma-3)\varepsilon-(\gamma+1)]}} \qquad (\gamma\neq 3) \tag{3.12}$$

$$\Theta_1^0=0, \quad \Theta_2^0=\frac{1}{2}\frac{\cot^2(\alpha/2)-2\ln(1-V_1)+1}{\sqrt{\cot^2(\alpha/2)-2\ln(1-V_1)}} \qquad (\gamma=3) \tag{3.13}$$

Setting

$$z^2=\begin{cases}\varepsilon-(\gamma+1)/(\gamma-3) & \text{for } \gamma\neq 3\\ \cot^2(\alpha/2)-2\ln(1-V_1) & \text{for } \gamma=3\end{cases}$$

and $z>0$ for Θ_2°, independently of γ we have

$$\Theta_2^\circ={}^1/_2(z+z^{-1}) \tag{3.14}$$

We now show that

$$z>1 \quad \text{for } 0<V_1\leqslant\frac{2}{\gamma-1}$$

For the case $\gamma=3$ this follows directly from the formula for z^2, since $\alpha\leq\frac{1}{2}\pi$.

For $\gamma\neq 3$ we consider two possibilities:

$$1<\gamma<3 \quad \text{for} \quad 0\leqslant\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}<1$$

We have

$$z^2=\frac{\gamma+1}{3-\gamma}\left[1-\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}\right]+\cot^2\frac{\alpha}{2}\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}>$$

$$>\frac{\gamma+1}{3-\gamma}+\left(1-\frac{\gamma+1}{3-\gamma}\right)\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}>\frac{\gamma+1}{3-\gamma}+2\frac{1-\gamma}{3-\gamma}=1$$

In the second case

$$\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}>1$$

We have

$$z^2=\frac{\gamma+1}{\gamma-3}\left[\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}-1\right]+\cot^2\frac{\alpha}{2}\left(1-\frac{\gamma-1}{2}V_1\right)^{\frac{3-\gamma}{\gamma-1}}>\cot^2\frac{\alpha}{2}\geqslant 1$$

From (3.14) for $z>1$ it follows that $\Theta_2^\circ>1$, so that inequality (2.8) is proved. Hence, we can always compute the mixed problem in regions of the type (3) touching the points C and C_1, since the hyperblicity of Equation (2.2) has been proved. Here the straight line DC separates the steady flow region of the type (1) from the simple wave region $SCDd$.

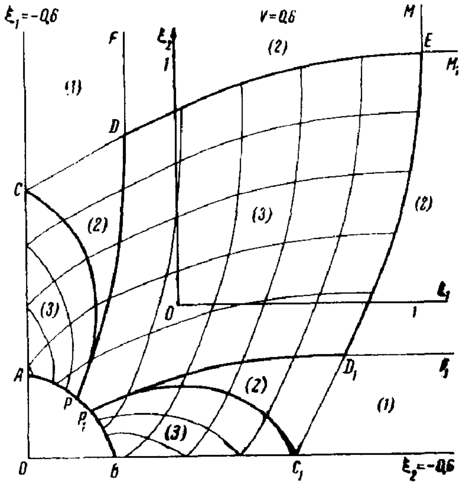

Fig. 3

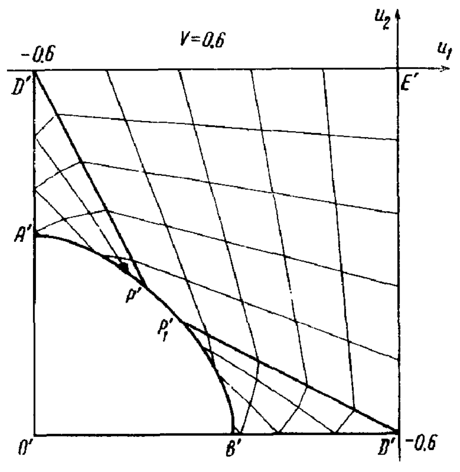

Fig. 4

The determination of the subsequent configuration of the flow region depends substantially on the actual values of V_1, α, and γ. It appears that such determination can only be effected numerically, the flow construction algorithm requiring solution of Goursat problems and mixed problems (near the walls), as well the construction of simple waves, It turns out that for fixed α and γ in the square $[0,2/(\gamma-1)]\times[0,2/(\gamma-1)]$ of the plane V_1, V_2 there always exists a curve which passes through the points $(0,2/(\gamma-1),0)$ and separates the square into two regions. For V_1 belonging to one of these regions there always arises a vacuum zone, and, as calculations show, Equation (2.2) remains hyperbolic. On the line of contiguity with the vacuum zone ($\Theta=0$), the functions Θ_1 and Θ_2 become infinite, but in such a way that $\Theta\Theta_1$ remains. The other region is characterized by undetached flow, but in the neighborhood of the point O there appears a line of parabolicity of Equation (2.2), and determination of the entire flow requires solution of a boundary value problem with a portion of the data on the degeneracy (parabolicity) line for Equation (2.2).

Fig. 2 shows the domain of definition of the flow in the hodograph plane. The lines $D'E'$, $D_1'E'$, $d'D'$, $d_1'D_1'$ correspond to simple waves, the Goursat problem must be solved in the region $d'D'E'D_1'd_1'$, and mixed problems in the regions $O'D'd'$ and $O'D_1'd_1'$.

4. Given below are the results of actual computations for certain values of the parameters α, γ, and V_1 (in the region of hyperbolicity of Equation (2.2)). The Goursat and mixed proplems were solved by the method of Massot characteristics with the iterations performed on a computer. As a rule,

30-40 computation points were taken along each characteristic. The applied program made it possible to carry out "straight through" computation of the configurations all the way to the line of contiguity with the vacuum zone or to the parabolicity line.

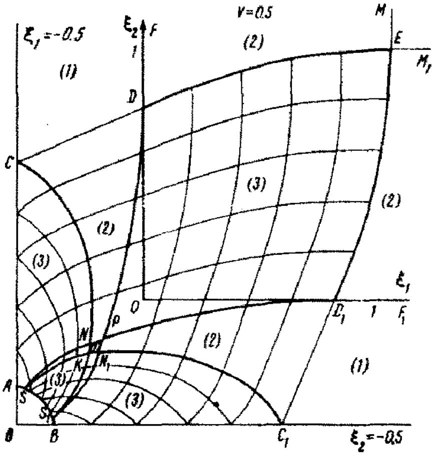

Fig.5

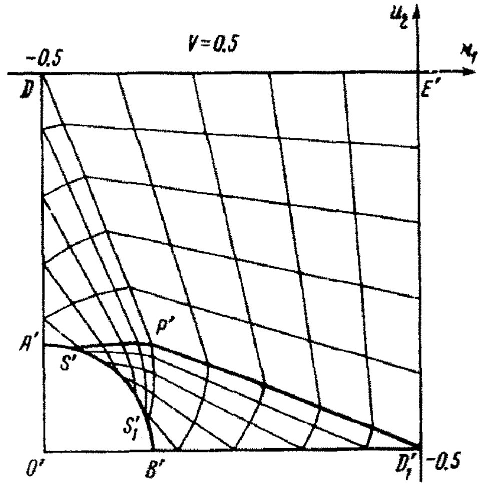

Fig.6

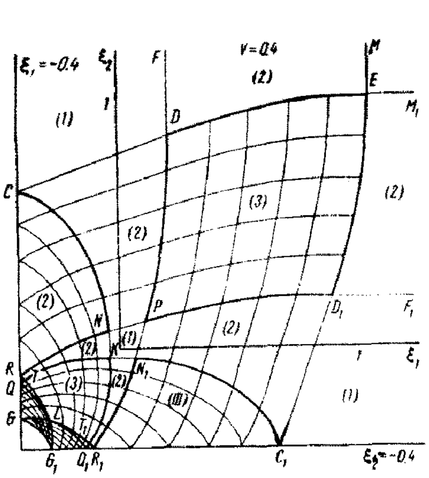

Fig.7

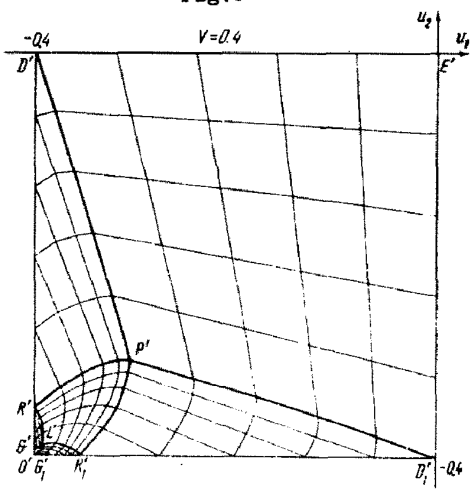

Fig.8

Figs. 3,5,and 7 show the configurations of the flow region in the coordinates ξ_1, ξ_2 and indicate the behavior of the characteristics of the system of equations in self-similar variables which describes the given motion for the cases $\alpha=\Pi/2, \gamma=3$, $V_1=V_2=V=0.6, 0.5, 0.4$. The regions of steady flow simple waves, and double waves are denoted by the numbers (1),(2), and (3),respectively. For the cases $V=0.6$, 0.5, a vacuum zone arises in the neighborhood of the point O: specifically, $\Theta=0$ on the line AB. The regions NSK and N_1S_1K in Fig.5 are of the type (2).

The case where $V=0.4$ is an example of undetached flow. The line GG_1 in Fig.7 is a parabolicity line; the characteristics in the region GLG_1 touch

GG_1 (in the plane ξ_1, ξ_2). The regions Q_1TR and $Q_1T_1R_1$ are of the type (1), the regions TQL and T_1Q_1L of the type (2), and QLG, Q_1LG_1, GLG_1 of the type (3). The function Θ changes little along GG_1 and is equal to approximately 0.12. The critical velocity $V_1 = V_2 = V^*$, which separates the cases of the appearance of a vacuum zone and the appearance of a parabolicity line is equal to 0.42 for the given α and γ.

Fig.4,6,8 also show the flow regions and characteristics in the hodograph plane for $V = 0.6, 0.5, 0.4$. The points in the hodograph plane corresponding to the points of the plane ξ_1, ξ_2 are accompanied by primes. The lines $D'P'$, $D_1'P'$, $D_1'P'$, $P'S'$, $P'S_1$, $P'R'$, $P'R_1'$, $L'R'$, $L'R_1'$ correspond to simple waves.

The region $O'G'L'G_1'$ and the line of parabolicity $G'H'G_1'$ are shown on a magnified scale in the hodograph plane in Fig.9. The boundary-value problem for Equation (2.2) must be solved in the region $O'G'H'G_1'$. As computations show the region of ellipticity of Equation (2.2) in the plane u_1, u_2 is very small.

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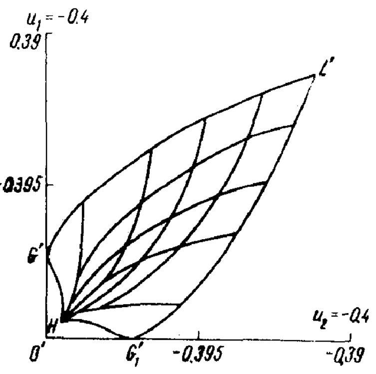

Fig.9

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